does not refer to maximum propagation speed of the interactions such as some authors suggest; c is just a speed limit which remains the same in any inertial frame.

The temporal coordinate x^0 of space-time is now $x^0 = V_{max}t$ ($x^0 = ct$ is then obtained when $V_{max} \rightarrow c$). Substitution of $V_{max} = nV = n\left(\tilde{H}l\right)$ into this equation yields $t = x^0 / V_{max} = \left(1/n\tilde{H}\right)\left(x^0/l\right)$. On the other hand, since $V = \tilde{H}l$ and $V = V_{max}/n$ we can write that $l = V_{max}\tilde{H}^{-1}/n$. Thus $\left(x^0/l\right) = \tilde{H}(nt) = \tilde{H}t_{max}$. Therefore we can finally write

$$t = \left(1/n\tilde{H}\right)\left(x^0/l\right) = t_{max}/n \qquad (37)$$

which shows the quantization of *time*.

From Eqs.(27) and (37) we can easily conclude that the *spacetime is not continuous* it is *quantized*.

Now, let us go back to Eq. (20) which will be called the *gravitational* Hamiltonian to distinguish it from the *inertial* Hamiltonian H_i:

$$H_i = c\sqrt{p^2 + m_{i0}{}^2c^2}\,. \qquad (38)$$

Consequently, the Eq. (18) can be rewritten in the following form:

$$H_i - H_g = 2\Delta H_i \qquad (39)$$

where ΔH_i is the *variation on the inertial Hamiltonian* or *inertial kinetic energy*. A *momentum* variation Δp yields a variation ΔH_i given by:

$$\Delta H_i = \sqrt{(p+\Delta p)^2c^2 + m_{i0}{}^2c^4} - \sqrt{p^2c^2 + m_{i0}{}^2c^4} \qquad (40)$$

By considering that the particle is *initially at rest* $(p = 0)$. Then Eqs. (20), (38) and (39) give respectively: $H_g = m_gc^2$, $H_i = m_{i0}c^2$ and

$$\Delta H_i = \left[\sqrt{1+\left(\frac{\Delta p}{m_{i0}c}\right)^2} - 1\right]m_{i0}c^2$$

By substituting H_g, H_i and ΔH_i into Eq.(39) we get

$$m_g = m_{i0} - 2\left[\sqrt{1+\left(\frac{\Delta p}{m_{i0}c}\right)^2} - 1\right]m_{i0}\,. \qquad (41)$$

This is the *general expression of correlation between the gravitational and inertial mass.* Note that for $\Delta p > m_{i0}c\left(\sqrt{5}/2\right)$, the value of m_g becomes *negative*.

Equation (41) shows that m_g decreases of Δm_g for an increase of Δp. Thus, starting from (4) we obtain

$$p + \Delta p = \frac{\left(m_g - \Delta m_g\right)V}{\sqrt{1-(V/c)^2}}$$

By considering that the particle is *initially at rest* $(p = 0)$, the equation above gives

$$\Delta p = \frac{\left(m_g - \Delta m_g\right)V}{\sqrt{1-(V/c)^2}}$$

From the Eq.(16) we obtain: $E_g = 2E_{i0} - E_i = 2E_{i0} - \left(E_{i0} + \Delta E_i\right) = E_{i0} - \Delta E_i$ However, Eq.(14) tells us that $-\Delta E_i = \Delta E_g$; what leads to $E_g = E_{i0} + \Delta E_g$ or $m_g = m_{i0} + \Delta m_g$. Thus, in the expression of Δp we can replace $\left(m_g - \Delta m_g\right)$ by m_{i0}, i.e.,

$$\Delta p = \frac{m_{i0}V}{\sqrt{1-(V/c)^2}}$$

We can therefore write

$$\frac{\Delta p}{m_{i0}c} = \frac{V/c}{\sqrt{1-(V/c)^2}} \qquad (42)$$

By substitution of the expression above into Eq. (41) we thus obtain:

$$m_g = m_{i0} - 2\left[\left(1 - V^2/c^2\right)^{-\frac{1}{2}} - 1\right]m_{i0} \qquad (43)$$

For $V = 0$ we obtain $m_g = m_{i0}$. Then

$$m_{g(min)} = m_{i0(min)}$$

Substitution of $m_{g(min)}$ into the *quantized* expression of M_g (Eq. (33)) gives

$$M_g = n^2 m_{i0(min)}$$

Where $m_{i0(min)}$ is the *elementary*

quantum of inertial mass to be determined.

For $V = 0$, the *relativistic* expression $M_g = m_g \big/ \sqrt{1-V^2/c^2}$ becomes $M_g = M_{g0} = m_{g0}$. However (43) shows that $m_{g0} = m_{i0}$. Thus, the *quantized* expression of M_g reduces to

$$m_{i0} = n^2 m_{i0(min)}$$

Then, we can write

$$\frac{m_{i0}}{\sqrt{1-V^2/c^2}} = n^2 \frac{m_{i0(min)}}{\sqrt{1-V^2/c^2}}$$

or

$$M_i = n_i^2 m_{i0(min)} \qquad (44)$$

Which shows the quantization of *inertial mass* ; n_i is the *inertial quantum number* .

We will change n in the quantized expression of M_g by n_g in order to define the *gravitational quantum number*. Thus we have

$$M_g = n_g^2 m_{i0(min)} \qquad (44a)$$

Finally, by substituting m_g given by Eq. (43) into the relativistic expression of M_g , we readily obtain

$$M_g = \left|\frac{m_g}{\sqrt{1-V^2/c^2}}\right| =$$

$$= \left| M_i - 2\left[\left(1-V^2/c^2\right)^{-\frac{1}{2}} - 1\right] M_i \right| \qquad (45)$$

The *Lorentz's force* is usually written in the following form:

$$d\,\vec{p}/dt = q\vec{E} + q\vec{V} \times \vec{B}$$

where $\vec{p} = m_{i0}\vec{V} \big/ \sqrt{1-V^2/c^2}$. However, Eq.(4) tells us that $\vec{p} = m_g \vec{V} \big/ \sqrt{1-V^2/c^2}$. Therefore, the expressions above must be corrected by multiplying its members by m_g / m_{i0} ,i.e.,

$$\vec{p}\,\frac{m_g}{m_{i0}} = \frac{m_g}{m_{i0}} \frac{m_{i0}\vec{V}}{\sqrt{1-V^2/c^2}} = \frac{m_g \vec{V}}{\sqrt{1-V^2/c^2}} = \vec{p}$$

and

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\vec{p}\,\frac{m_g}{m_{i0}}\right) = \left(q\vec{E} + q\vec{V} \times \vec{B}\right)\frac{m_g}{m_{i0}} \qquad (46)$$

That is now the *general expression* for Lorentz's force. Note that it depends on m_g .

When the force is perpendicular to the speed, the Eq.(5) gives $d\vec{p}\big/dt = m_g\left(d\vec{V}\big/dt\right)\big/\sqrt{1-V^2/c^2}$ .By comparing with Eq.(46) we thus obtain

$$\left(m_{i0}\big/\sqrt{1-V^2/c^2}\right)\left(d\vec{V}/dt\right) = q\vec{E} + q\vec{V} \times \vec{B}$$

Note that this equation is the expression of an *inertial* force.

Starting from this equation, well-known experiments have been carried out in order to verify the relativistic expression: $m_i \big/ \sqrt{1-V^2/c^2}$. In general, the *momentum* variation Δp is expressed by $\Delta p = F\Delta t$ where F is the applied force during a time interval Δt . Note that there is no restriction concerning the *nature* of the force F , i.e., it can be mechanical, electromagnetic, etc.

For example, we can look on the *momentum* variation Δp as due to absorption or emission of *electromagnetic energy* by the particle (by means of *radiation* and/or by means of *Lorentz's force* upon the *charge* of the particle).

In the case of radiation (any type), Δp can be obtained as follows. It is known that the *radiation pressure* , dP , upon an area $dA = dxdy$ of a volume $dV = dxdydz$ of a particle(the incident radiation normal to the surface dA)is equal to the energy dU absorbed per unit volume (dU/dV).i.e.,

$$dP = \frac{dU}{dV} = \frac{dU}{dxdydz} = \frac{dU}{dAdz} \qquad (47)$$

Substitution of $dz = vdt$ (v is the speed of radiation) into the equation above gives