

energy E and momentum \vec{p} , moving in direction $+x$.

As to the imaginary particle, the *imaginary particle wave function* will be denoted by Ψ_ψ and, by analogy with the expression of Ψ , will be expressed by:

$$\Psi_\psi = \Psi_{0\psi} e^{-(2\pi i/h)(E_\psi t - p_\psi x)}$$

Therefore, the *general expression* of the wave function for a *free particle* can be written in the following form

$$\Psi = \Psi_{0(\text{real})} e^{-(2\pi i/h)(E_{(\text{real})} t - p_{(\text{real})} x)} + \Psi_{0\psi} e^{-(2\pi i/h)(E_\psi t - p_\psi x)}$$

It is known that the *uncertainty principle* can also be written as a function of ΔE (uncertainty in the energy) and Δt (uncertainty in the time), i.e.,

$$\Delta E \cdot \Delta t \geq \hbar$$

This expression shows that a variation of energy ΔE , during a time interval Δt , can only be detected if $\Delta t \geq \hbar/\Delta E$. Consequently, a variation of energy ΔE , during a time interval $\Delta t < \hbar/\Delta E$, cannot be experimentally detected. This is a limitation imposed by Nature and not by our equipments.

Thus, a *quantum* of energy $\Delta E = hf$ that varies during a time interval $\Delta t = 1/f = \lambda/c < \hbar/\Delta E$ (wave period) cannot be experimentally detected. This is an *imaginary* photon or a “*virtual*” photon.

Now, consider a particle with energy $M_g c^2$. The DeBroglie’s gravitational and inertial wavelengths are respectively $\lambda_g = h/M_g c$ and $\lambda_i = h/M_i c$. In Quantum Mechanics, particles of matter and quanta of radiation are described by means of *wave packet* (DeBroglie’s waves) with average wavelength λ_i .

Therefore, we can say that during a time interval $\Delta t = \lambda_i/c$, a *quantum* of energy $\Delta E = M_g c^2$ varies. According to the uncertainty principle, the particle will be detected if $\Delta t \geq \hbar/\Delta E$, i.e., if $\lambda_i/c \geq \hbar/M_g c^2$ or $\lambda_i \geq \lambda_g/2\pi$. This condition is usually satisfied when $M_g = M_i$. In this case, $\lambda_g = \lambda_i$ and obviously, $\lambda_i > \lambda_i/2\pi$. However, when M_g decreases λ_g increases and $\lambda_g/2\pi$ can become greater than λ_i , making the particle *non-detectable* or *imaginary*.

According to Eqs. (7) and (41) we can write M_g in the following form:

$$M_g = \left| \frac{m_g}{\sqrt{1 - V^2/c^2}} \right| = \left| \frac{\chi m_i}{\sqrt{1 - V^2/c^2}} \right| = |\chi| M_i$$

Where

$$\chi = \left\{ 1 - 2 \left[\sqrt{1 + (\Delta p/m_{i0} c)^2} - 1 \right] \right\}$$

Since the condition to make the particle *imaginary* is

$$\lambda_i < \frac{\lambda_g}{2\pi}$$

and

$$\frac{\lambda_g}{2\pi} = \frac{\hbar}{M_g c} = \frac{\hbar}{|\chi| M_i c} = \frac{\lambda_i}{2\pi |\chi|}$$

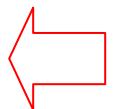
Then we get

$$|\chi| < \frac{1}{2\pi} = 0.159$$

This means that when

$$-0.159 < \chi < +0.159$$

The particle becomes *imaginary*. Under these circumstances, we can say that **the particle made a transition to the imaginary space-time.**



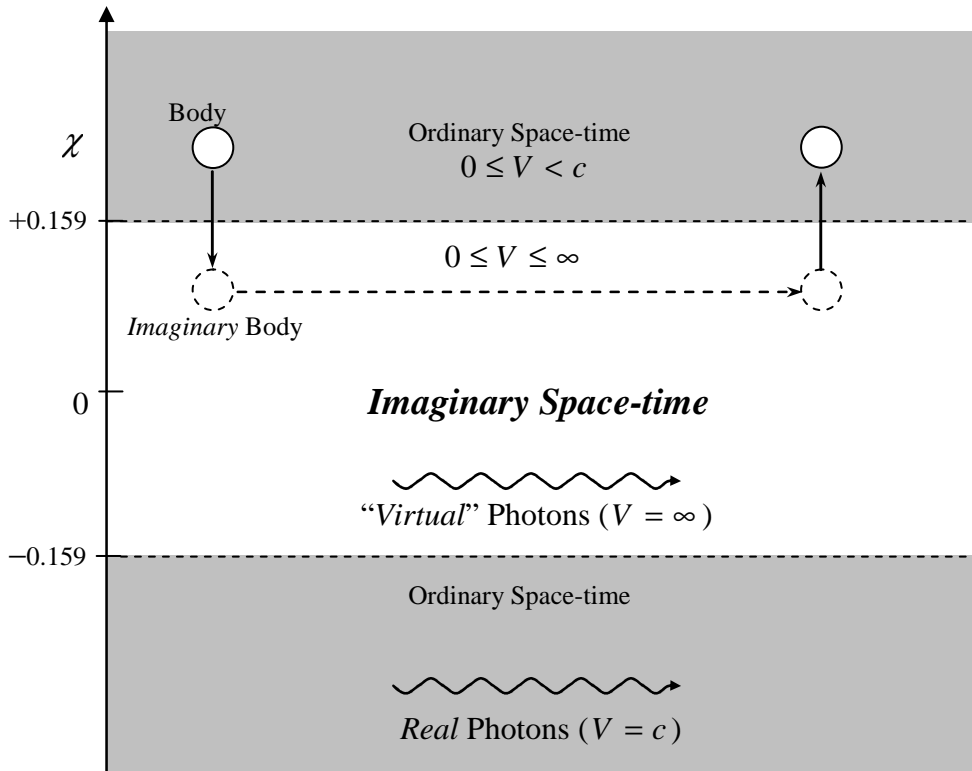


Fig. VII – *Travel in the imaginary space-time*. Similarly to the “virtual” photons, imaginary bodies can have infinite speed in the imaginary space-time.

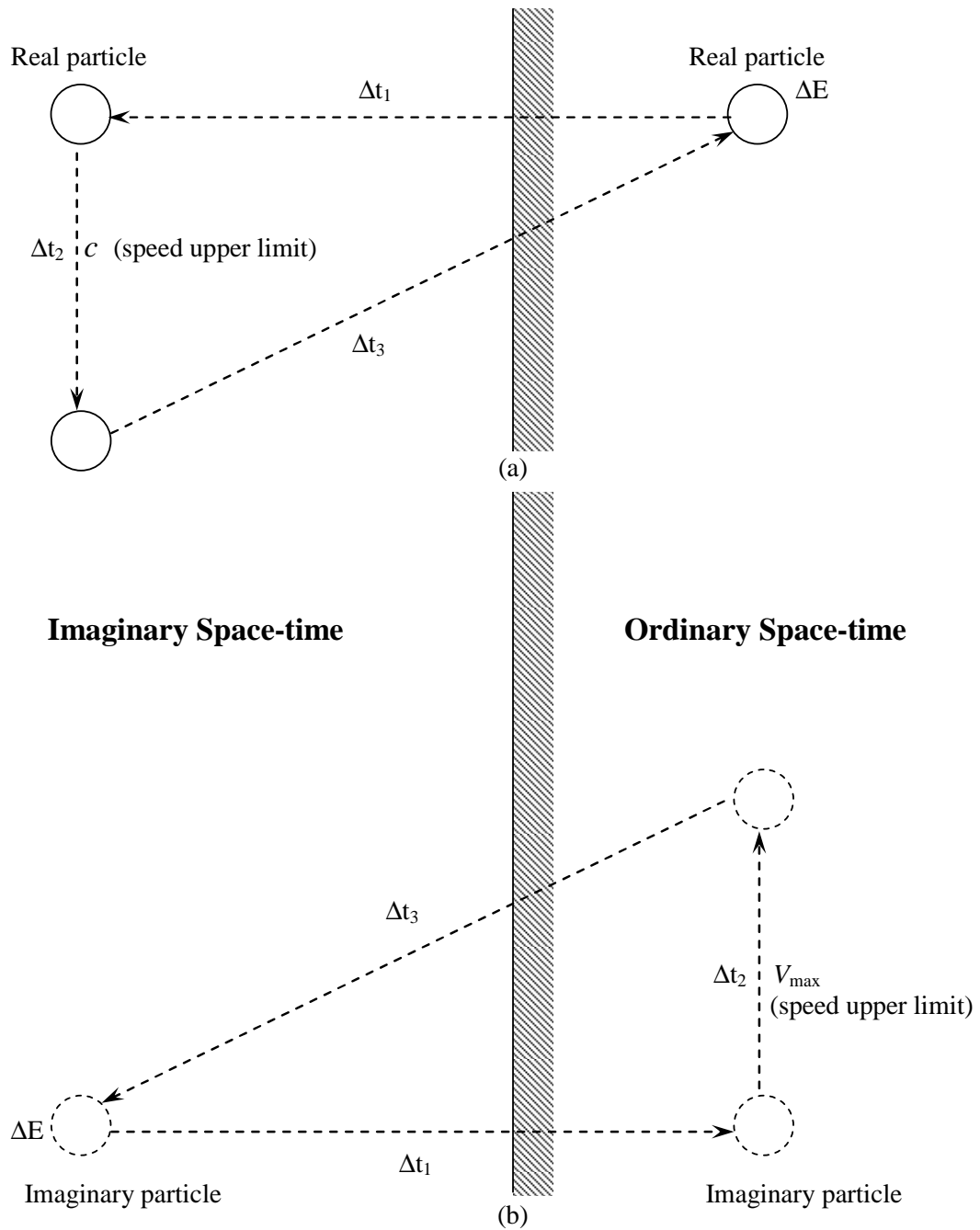


Fig. VIII – “Virtual” Transitions – (a) “Virtual” Transitions of a *real* particle to the *imaginary* space-time. The speed upper limit for *real* particle in the *imaginary* space-time is c . (b) - “Virtual” Transitions of an *imaginary* particle to the *ordinary* space-time. The speed upper limit for *imaginary* particle in the *ordinary* space-time is $V_{max} \approx 10^2 m.s^{-1}$
 Note that to occur a “virtual” transition it is necessary that $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3 < \hbar / \Delta E$
 Thus, even at principle, it will be impossible to determine any variation of energy in the particle (*uncertainty principle*).

Note that, when a particle becomes imaginary, its gravitational and inertial masses also become imaginary. However, the factor $\chi = M_{g(\text{imaginary})}/M_{i(\text{imaginary})}$ remains *real* because

$$\chi = \frac{M_{g(\text{imaginary})}}{M_{i(\text{imaginary})}} = \frac{M_g i}{M_i i} = \frac{M_g}{M_i} = \text{real}$$

Thus, if the gravitational mass of the particle is reduced by means of the absorption of an amount of electromagnetic energy U , for example, we have

$$\chi = \frac{M_g}{M_i} = \left\{ 1 - 2 \left[\sqrt{1 + (U/m_{i0}c^2)^2} - 1 \right] \right\}$$

This shows that the energy U of the electromagnetic field *remains acting* on the imaginary particle. In practice, this means that *electromagnetic fields act on imaginary particles*.

The gravity acceleration on a *imaginary* particle (due to the rest of the imaginary Universe) are given by

$$g'_j = \chi g_j \quad j = 1, 2, 3, \dots, n.$$

Where $\chi = M_{g(\text{imaginary})}/M_{i(\text{imaginary})}$

and $g_j = -Gm_{gj(\text{imaginary})}/r_j^2$. Thus, the gravitational forces acting on the particle are given by

$$\begin{aligned} F_{gj} &= M_{g(\text{imaginary})} g'_j = \\ &= M_{g(\text{imaginary})} \left(-\chi G m_{gj(\text{imaginary})} / r_j^2 \right) = \\ &= M_g i \left(-\chi G m_{gj} i / r_j^2 \right) = + \chi G M_g m_{gj} / r_j^2. \end{aligned}$$

Note that these forces are *real*. Remind that, the Mach's principle says that the *inertial effects* upon a particle are consequence of the gravitational interaction of the particle with the rest of the Universe. **Then we can conclude that the inertial forces upon an imaginary particle are also real.**

Equation (7) shows that, in the case of imaginary particles, the relativistic mass is

$$\begin{aligned} M_{g(\text{imaginary})} &= \frac{m_{g(\text{imaginary})}}{\sqrt{1 - V^2/c^2}} = \\ &= \frac{m_g i}{i \sqrt{V^2/c^2 - 1}} = \frac{m_g}{\sqrt{V^2/c^2 - 1}} \end{aligned}$$

This expression shows that *imaginary* particles can have velocities V greater than c in our ordinary space-time (Tachyons). The *quantization of velocity* (Eq. 36) shows that there is a speed upper limit $V_{max} > c$. As we have already calculated previously, $V_{max} \approx 10^{12} m.s^{-1}$, (Eq.102).

Note that this is the speed upper limit for imaginary particles in our ordinary space-time not in the imaginary space-time (Fig.7) because the infinite speed of the "virtual" quanta of the interactions shows that imaginary particles can have infinite speed in the imaginary space-time.

While the speed upper limit for imaginary particles in the ordinary space-time is $V_{max} \approx 10^{12} m.s^{-1}$, the speed upper limit for *real* particles in the *imaginary* space-time is c , because the relativistic expression of the mass shows that the velocity of *real* particles cannot be greater than c in *any space-time*. The uncertainty principle permits that particles make "virtual" transitions, during a time interval Δt , if $\Delta t < \hbar/\Delta E$. The "virtual" transition of *mesons* emitted from nucleons that does not change of mass, during a time interval $\Delta t < \hbar/m_\pi c^2$, is a well-known example of "virtual" transition of